1. Write the degree of each of the following polynomials

(i) $5x^3+4x^2+7x$ (ii) $4-y^2$ (iii) $5t-\sqrt{7}$ (iv) 3

Sol. **(i)** The degree of $5x^3+4x^2+7x$ is 3.

(ii) The degree of $4-y^2$ is 2.

(iii) The degree of $5t-\sqrt{7}=5t^1-\sqrt{7}$ is 1.

(iv) The degree of $3=3x^0$ is 0.

2. Classify the following as linear, quadratic and cubic polynomials:

(i) x^2+x (ii) $x-x^3$ (iii) $y+y^2+4$ (iv) $1+x$

(v) $3t$ (vi) r^2 (vii) $7x^3$

Sol.

(i) x^2+x Quadratic polynomial.

(ii) $x-x^3$ Cubic polynomial.

(iii) $y+y^2+4$ Quadratic polynomial.

(iv) $1+x$ Linear polynomial.

(v) $3t$ Linear polynomial.

(vi) r^2 Quadratic polynomial.

(vii) $7x^3$ Cubic polynomial.

3. Find the value of the polynomial $5x-4x^2+3$ at

(i) $x=0$ (ii) $x=-1$ (iii) $x=2$

Sol. **(i)** Putting $x=0$, we get

$p(0)=5\times0-4(0)^2+3=3$

(ii) Putting $x=-1$, we get

$p(-1)=5\times(-1)-4(-1)^2+3=-5-4+3=-6$

(iii) Putting $x=2$, we get

$p(2)=5\times2-4(2)^2+3=10-16+3=-3$

4. Find the remainder when x^3+3x^2+3x+1 is divided by

(i) $x+1$ (ii) $x-\frac{1}{2}$ (iii) x (iv) $x+\pi$

(v) $5+2x$

Sol. Let $p(x)=x^3+3x^2+3x+1$

(i) $x+1$

Putting $x+1=0$, **we get** $x=-1$

Using remainder theorem, when $p(x)=x^3+3x^2+3x+1$ is divided by $x+1$, remainder is given by $p(-1)$

$=(-1)^3+3(-1)^2+3(-1)+1$

$=-1+3-3+1$

= 0

(ii) $x-\frac{1}{2}$

Putting $x-\frac{1}{2}=0$, **we get,** $x=\frac{1}{2}$

Using remainder theorem, when $p(x)=x^3+3x^2+3x+1$ divided by $x-\frac{1}{2}$, remainder is given by $p\left(\frac{1}{2}\right)$

$=\left(\frac{1}{2}\right)^3+3\left(\frac{1}{2}\right)^2+3\left(\frac{1}{2}\right)+1$

$=\frac{1}{8}+3\times\frac{1}{4}+3\times\frac{1}{2}+1$

$=\frac{1+6+12+8}{8}$

$=\frac{27}{8}$

(iii) x

Putting $x=0$, we get

Using remainder theorem, when $p(x)=x^3+3x^2+3x+1$ is divided by x, remainder is given by $p(0)$

$=(0)^3+3(0)^3+3(0)+1$

$=0+1$

$=0$

(iv) $x+\pi$

Putting $x+\pi=0$, we get, $x=-\pi$

Using remainder theorem, when $p(x)=x^3+3x^2+3x+1$ is divided by $x+\pi$, remainder is given by $p(-\pi)$

$=(-\pi)^3+3(-\pi)^2+3(-\pi)+1$

$=-\pi^3+3\pi^2-3\pi+1$

(v) $5+2x$

Putting $5+2x=0,$ we get $x=-\frac{5}{2}$

Using remainder theorem, when $p(x)=x^3+3x^2+3x+1$ is divided by $5+2x$, remainder is given by $p\left(-\frac{5}{2}\right)$

$$=\left(-\frac{5}{2}\right)^3+3\left(-\frac{5}{2}\right)^2+3\left(-\frac{5}{2}\right)+1$$

$$=-\frac{125}{8}+3\times\frac{25}{4}-3\times\frac{5}{2}+1$$

$$=\frac{-125+150-60+8}{8}$$

$$=-\frac{27}{8}$$

5. Find the remainder when x^3-ax^2+6x-a is divided by $x-a$.

Sol. Let $p(x)=x^3-ax^2+6x-a$

Putting $x-a=0,$ we get $x=a$

Using remainder theorem, when $p(x)=x^3-ax^2+6x-a$ is divided by $x-a,$ remainder is given by $p(a)$

$$=(a)^3-a(a)^2+6(a)-a$$

$$=a^3-a^3+6a-a$$

$$=5a$$

6. Find the value of k, if $x-1$ is a factor of $p(\text{x})$ in each of the following cases:

(i) $p(x)=x^2+x+k$

(ii) $p(x)=2x^2+kx+\sqrt{2}$

(iii) $p(x)=kx^2-\sqrt{2}x+1$

(iv) $p(x)=kx^2-3x+k$

Sol. **(i)** If $(x-1)$ is a factor of $p(x)=x^2+x+k,$ then

$p(1)=0$

$\Rightarrow \quad (1)^2+1+k=0$

$\Rightarrow \quad 1+1+k=0 \qquad \Rightarrow k=-2$

(ii) If $(x-1)$ is a factor of $p(x)=2x^2+kx+\sqrt{2}$, then

$p(1)=0$

$\Rightarrow 2(1)^2+k(1)+\sqrt{2}=0 \quad \Rightarrow 2+k+\sqrt{2}=0$

$\Rightarrow k=-(2+\sqrt{2})$

(iii) If $(x-1)$ is a factor of $p(x)=kx^2-\sqrt{2}x+1$, then

$p(1)=0$

$\Rightarrow k(1)^2-\sqrt{2}(1)+1=0 \quad \Rightarrow k-\sqrt{2}+1=0$

$\Rightarrow k=\sqrt{2}-1$

(iv) If $(x-1)$ is a factor of $p(x)=kx^2-3x+k$, then

$p(1)=0$

$\Rightarrow k(1)^2-3(1)+k=0 \quad \Rightarrow k-3+k=0$

$2k=3 \quad \Rightarrow k=\dfrac{3}{2}$

7. Factorise the following using appropriate identities:

(i) $9x^2+6xy+y^2$ (ii) $4y^2-4y+1$ (iii) $x^2-\dfrac{y^2}{100}$

Sol. **(i)** $9x^2+6xy+y^2=(3x)^2+2(3x)(y)+(y)^2$

$=(3x+y)^2=(3x+y)(3x+y)$

(ii) $4y^2-4y+1=(2y)^2-2(2y)(1)+(1)^2$

$=(2y-1)^2=(2y-1)(2y-1)$

(iii) $x^2-\dfrac{y^2}{100}=(x)^2-\left(\dfrac{y}{10}\right)^2$

$=\left(x-\dfrac{y}{10}\right)\left(x+\dfrac{y}{10}\right)$

8. Factorise each of the following:

(i) $27y^3+125z^3$ (ii) $64m^3-343n^3$

Sol. **(i)** $27y^3+125z^3=(3y)^3+(5z)^3$

$=(3y+5z)[(3y)^2-(3y)(5z)+(5z)^2]$

$=(3y+5z)(9y^2-15yz+25z^2)$

(ii) $64m^3-343n^3=(4m)^3-(7n)^3$

$= (4m-7n)[(4m)^2+(4m)(7n)+(7n)^2]$

$= (4m-7n)[16\text{m}^2+28mn+49n^2]$

9. Factorise:

(i) $27x^3+y^3+z^3-9xyz$

Sol. $27x^3+y^3+z^3-9xyz = (3x)^3+y^3+z^3-3(3\text{x})(\text{y})(\text{z})$

$= (3x+y+z)[(3x)^2+y^2+z^2-(3x)y-yz-z(3x)]$

$= (3x+y+z)\ (9x^2+y^2+z^2-3xy-yz-3zx)$

10. If $x+y+z=0,$ show that $x^3+y^3+z^3=3xyz,$

Sol. We know that,

$x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$= 0(x^2+y^2+z^2-xy-yz-zx)$ $(\because x+y+z=0 \text{ given})$

$= 0$

$\Rightarrow x^3+y^3+z^3=3xyz$ Hence proved.